ANALYSIS OF REAL BUSINESS CYCLE MODELS WITH CAPITAL-SKILL COMPLEMENTARITY

A Master's Thesis

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To My Family

ANALYSIS OF REAL BUSINESS CYCLE MODELS WITH CAPITAL-SKILL COMPLEMENTARITY

The Institute of Economics and Social Sciences
of
Bilkent University

by

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In Partial Fulfillment of the Requirements For the Degree of
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in

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January 2009

I certify that I have read this thesis and have found that it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Arts in Economics.

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ABSTRACT

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January 2009

The aim of this paper is to analyze the effects of a production function with capital-skill complementarity, as opposed to the standard Cobb-Douglas production function, on Real Business Cycle [RBC] models. Capital-skill complementarity is mostly used in open economy models to study the wage inequality but this production function has not been employed in closed economy real business cycle models. This paper shows that the production function with capital-skill complementarity causes even lower endogenous propagation of the impulse response functions than those of standard closed economy RBC models, which already have problems with regards to the persistence of output.

Keywords: Capital-Skill Complementarity, Real Business Cycle Models, Persistence, Adjustment Cost.

ÖZET

SERMAYE-VASIFLI İŞGÜCÜ TAMAMLAYICILIĞI İÇEREN REEL İŞ ÇEVRİMLERİ MODELLERİNİN İNCELENMESİ

KILIÇ, Sinem

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Bu çalışmanın amacı, standart Cobb-Douglas üretim fonksiyonundan farklı olarak, sermaye-vasıflı işgücü tamamlayıcılığı olan bir üretim fonksiyonunun Reel İş Çevrimleri [RİÇ] modelleri üzerindeki etkilerini incelemektir. Sermaye-vasıflı işgücü tamamlayıcılığı genellikle açık ekonomi modellerinde ücret eşitsizliklerini analiz etmek için kullanılmaktadır. Bu üretim fonksiyonları kapalı ekonomi RİÇ modellerinde henüz kullanılmamıştır. Tezin sonuçları göstermektedir ki, sermaye-vasıflı işgücü tamamlayıcılığı olan üretim fonksiyonlarının kullanımı, halihazırda üretim üzerindeki etkinin sürerliği açısından sorun bulunan standart kapalı ekonomi RİÇ modellerine kıyasla daha da düşük içsel yansıma içeren uyarı yansıma fonksiyonlarına neden olmaktadır.

Anahtar Kelimeler: Sermaye-İşgücü Tamamlayıcılığı, Reel İş Çevrimleri Modelleri, Sürerlik, Düzeltme Maliyeti.

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CHAPTER 1

INTRODUCTION

Capital-skill complementarity, where physical capital and skilled labor are complements in the production function, is mostly used in open economy models to study the relationship between openness and wage inequality. However, the properties of this production function are not well known so it is worthwhile to analyze it in a context which is extensively studied. This paper studies the class of production functions exhibiting capital-skill complementarity in closed economy Real Business Cycle [RBC] models, of which we have a good understanding, on both theoretically and empirically.

The aim of the paper is to analyze the persistence of output in closed economy RBC models with and without capital-skill complementarity and to see whether extension of the classic RBC models with capital-skill complementarity makes any improvement on the endogenous propagation generated by the model.

Krusell *et al.* (2000) develop a framework that provides a simple, explicit economic mechanism to study the skill-biased technical change to evaluate the changes in skill premium by simply looking at the changes in factor quantities using capital-skill complementarity. They solve a simple model which focuses on the aggregate production function and explain the wage inequality in the US by capital-skill complementarity in a calibrated partial equilibrium framework. In an earlier paper, Stokey (1996) uses capital-skill complementarity in a neoclassical growth model with physical and human capital accumulation to study the wage inequality in the case of factor trade. She shows that in the small open economy case, inflows of physical capital increase the wage premium compared to the closed economy by virtue of the production function assumed.

While using capital-skill complementarity as a technology preference in theoretical models is new, the analysis of the business cycle fluctuations with general equilibrium models begins with the pioneering article of Kydland and Prescott in the early 1980's. Kydland and Prescott (1982) introduce the idea that business fluctuations can be studied using dynamic general equilibrium models and we can quantitatively compare the features of a model economy with stylized facts of real economy which is known from the empirical studies. The reason behind naming this line of literature as "real" business cycle is the fact that "real" shocks, in particular technology shocks, lead to the business fluctuations.

Kydland and Prescott and many others find that simulated data from the dynamic general equilibrium models exhibit patterns similar to the actual US data regarding the persistence, volatility and a comovement of aggregate variables.[1] However, these business cycles that fit the data are driven mainly by large and cyclically volatile shocks to productivity measured by Solow residuals (Prescott, 1986). The Solow residual is problematic since there is almost no macro shocks that produce productivity variations suggested by Solow residuals (King and Rebelo, 2000). In fact, one of the problems of the standard RBC models is that these models do not have a strong enough endogenous mechanism to propagate shocks over time (Cogley and Nason, 1995).

The study conducted by King and Rebelo (2000) also shows that the basic RBC model requires large technology shocks to produce realistic business cycles. They apply an HP filter to produce cyclical components for US aggre-

[1] See Long and Plosser (1983) and Prescott (1986).

Table 1: Persistence Coefficients in King & Rebelo (2000)

Variables	Output (y)	Consumption (c)	Investment (I)	Labor (l)	Wage (w)
AC for US data	0.84	0.80	0.87	0.88	0.66
AC for RBC model	0.72	0.79	0.71	0.71	0.76

gates using the data that covers the period from the first quarter of 1947 to the last quarter of 1996 and compare the features of these series with those generated by the calibration of the basic RBC model. Considering the persistence of the series of the macroeconomic aggregates, they compare the first order serial correlation coefficients and conclude that the persistence generated by the basic model is high but weaker than those obtained from the real data.[2] Moreover, Watson (1993) shows that basic RBC models fail to match properties of the spectral density of major economic dynamics as a result of weak internal persistence.

In the RBC literature there are many papers that try to increase the persistence of shocks. One way of propagating shocks is to extend the model by adding adjustment costs either to capital or to labor. In their paper, Cogley and Nason (1995) extend the basic RBC model by adding quadratic capital adjustment cost and quadratic costs of labor input and conclude that addition of only capital adjustment costs has almost no effect on the model's impulse response functions but adding both a capital and labor adjustment costs to the model accounts for serial correlation in output growth. Using an RBC model in which there is an adjustment cost of labor, Janko (2004) concludes that the adjustment cost introduces an endogenous propagation mechanism for both technology and consumption shocks. Moreover, Poveda (2003) displays that the dynamics of an RBC model with a risk averse firm without adjustment costs are the same with those obtained from a standard RBC model with cap-

[2]See Table 1.

ital adjustment cost. Finally, Benhabib *et al.* (2006) find that three-sector model has a strong propagation mechanism as long as the factor intensities in the three sectors are different enough.

In this study, I use capital-skill complementarity in a standard RBC model and ask whether the addition of capital-skill complementarity increases the endogenous propagation mechanism of the model. The paper is organized as follows. Next section firstly, outlines the models of single agent with Cobb-Douglas production function and the model of two agents with capital-skill complementarity, respectively and secondly, displays the extensions of these models by adding capital adjustment costs. Section 3 presents the parameters underlying the simulations of the models and displays the simulation results. Section 4 concludes.

CHAPTER 2

MODELS

2.1 Production Technology

The aim of this paper is to analyze the effect of capital-skill complementarity, where physical capital and skilled labor are complements in a particular type of production function. To do that, I follow main insight of Stokey (1996) having production function with three factors: skilled labor (n), unskilled labor (1-n), and physical capital (k):

$$f(k,n) = a[\theta k^v + (1-\theta)(1-n)^v]^{\alpha/v}[n + e(1-n)]^{1-\alpha}$$

In Stokey's model labor provides two distinct productive services, namely *physical effort* and *mental effort* and unskilled labor provides both services, while skilled labor supplies the latter. Thus, in this production function, the parameter e is important in the sense that it refers to relative efficiency of unskilled labor in supplying mental effort. In Stokey's model n is just a state variable, there is no choice of leisure and the sum of skilled labor and unskilled labor is constant and it is normalized to one. Thus, e being equal to 0 leads to fixed skilled labor's share of total output at $1-\alpha$.

In this analysis, a more simplified version of this production function is used:

$$f(k,s,u) = a[\theta k^v + (1-\theta)u^v]^{\alpha/v}s^{1-\alpha}. \tag{2.1}$$

Here, there are also three factors: skilled labor (s), unskilled labor (u), and physical capital (k).

Similar to Stokey's model, two distinct productive services are provided by labor but this time skilled worker only provides skilled labor, mental effort in Stokey's terminology, and unskilled worker only provides unskilled labor, namely physical effort. Hence, the relative efficiency of unskilled labor in supplying mental effort, e, is set to be equal 0. Unlike Stokey's model, although $e = 0$, the skilled labor's share of total output is not fixed since both unskilled labor, u, and skilled labor, s are choice variables.[1]

Before the details of the model, it is important to explain the parameters of this production function. a displays the technology level of the economy. θ is the factor share of the physical capital and v determines the elasticity of substitution between physical capital and unskilled labor, $\frac{1}{1-v}$.[2] Actually, if $\frac{1}{1-v} \geq 1$ then k and u are substitutes and they are both complementary to skilled labor, s.

Since the aim of this paper is to analyze the effect of capital-skill complementarity on RBC models, I will first introduce the very basic RBC model with Cobb-Douglas production function and then, I will analyze the model having capital-skill complementarity. Later, I will add capital adjustment cost to both models to see whether this extension makes any difference in the comparison of the models.

2.2 Single Agent RBC Model with Cobb-Douglas Technology

Household's Problem:

[1]Different than Stokey's social planner problem, utility maximization problem of both skilled and unskilled labor also depends on their leisure, which are denoted by $1 - s$ and $1 - u$, respectively.

[2]In line with the definition of Allen-Uzawa partial elasticity of substitution, $\frac{1}{1-v}$ is the intraclass elasticity of substitution within class of physical capital and unskilled labor.

The representative household has factors of labor (l) and physical capital (k). She tries to maximize the expected discounted utility depending on her consumption and hours worked as she gets utility from leisure (1-l). She also owns the physical capital and decides how much invest on it.

$$max\ E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{c_t^{1-\rho_1}}{1-\rho_1} + \psi \frac{(1-l_t)^{1-\rho_2}}{1-\rho_2} \right]$$

subject to

$$k_{t+1} = (1-\delta)k_t + I_t$$

$$c_t + I_t = f(k_t, l_t)$$

First order conditions of this problem can be expressed as in the following:

$$\psi \frac{c_t^{\rho_1}}{(1-l_t)^{\rho_2}} = w_t \tag{2.2}$$

$$\left(\frac{c_{t+1}}{c_t} \right)^{\rho_1} = \beta[1-\delta+r_{t+1}] \tag{2.3}$$

We can see from (2.2) that wage is equal to the ratio of consumption to leisure times how much household weights on leisure (ψ). This implies that there is a trade-off between consumption and leisure, which is determined by wage, inter-temporal elasticity of consumption, ρ_1 and that of leisure, ρ_2, and the weight of leisure in utility, ψ. Equation (2.3) is the usual Euler equation that determines the inter-temporal consumption decision, which depends on inter-temporal elasticity of consumption, discount rate, β, depreciation rate, δ, and interest rate, r.

Firm's Problem:

Assuming perfectly competitive market and specifying Cobb-Douglas technology as in the following form:[3]

[3]The standard Cobb-Douglas technology is a special type of production function that is used for capital skill complementarity, where $v = 1$ and $\theta = 1$ in (2.1).

$$f(k_t, l_t) = e^{z_t} a k_t^{\alpha} l_t^{1-\alpha}$$

where z_t is the technology shock that involves according to an AR(1) process:

$$z_t = \sigma z_{t-1} + \epsilon_t \tag{2.4}$$

The firm solves its profit maximization and this yields:

$$f_k(k_t, l_t) = r_t$$

$$f_l(k_t, l_t) = w_t$$

where

$$f_k(k_t, l_t) = \alpha \frac{f(k_t, l_t)}{k}$$
$$f_l(k_t, l_t) = (1-\alpha) \frac{f(k_t, l_t)}{l}$$

2.3 Two-Agent Model with Capital-Skill Complementarity

In this model there two different types of agents: One is skilled worker and the other is unskilled. Both skilled and unskilled workers try to maximize their utility depending on the consumption level and hours worked as they get utility from leisure. Skilled worker has two factors: skilled labor (s) and physical capital (k). Unskilled worker has no physical capital, she only has unskilled labor (u). Therefore, all of the physical capital is owned by skilled worker and she decides on how much invests on it.

Problem of Skilled Labor:

Having utility dependent on consumption, c^s, and leisure, $1 - s$, skilled worker tries to maximize her expected discounted utility subject to her budget constraint and physical capital accumulation.

$$max\ E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{c_t^{s(1-\rho_1)}}{1-\rho_1} + \psi \frac{(1-s_t)^{(1-\rho_2)}}{1-\rho_2} \right]$$

subject to

$$k_{t+1} = (1-\delta)k_t + I_t$$

$$c_t^s + I_t = w_t^s s_t + r_t k_t$$

First order conditions of this problem are as follows:

$$\psi \frac{(c_t^s)^{\rho_1}}{(1-s_t)^{\rho_2}} = w_t^s \tag{2.5}$$

$$\left(\frac{c_{t+1}^s}{c_t^s} \right)^{\rho_1} = \beta[r_{t+1} + (1-\delta)] \tag{2.6}$$

Equation (2.5) that there is a trade off between consumption of skilled worker and her leisure, $(1-s)$, which is determined by wage of skilled worker, inter-temporal elasticity of consumption and that of leisure and the weight of leisure in utility function. Moreover, Euler equation, (2.6), states that the dynamics of the consumption level are determined by interest rate, r, inter-temporal elasticity of consumption, ρ_1, discount rate, β, and the depreciation rate, δ.

Problem of Unskilled Labor:

Different from skilled labor, unskilled labor does not have physical capital. Thus, she maximizes her expected discounted utility subject to only budget constraint.

$$max\ E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{c_t^{u(1-\rho_1)}}{1-\rho_1} + \psi \frac{(1-u_t)^{(1-\rho_2)}}{1-\rho_2} \right]$$

subject to

$$c_t^u = w_t^u u_t$$

c^u is consumption of unskilled labor. First order conditions of the problem of unskilled labor are:

$$\left(\frac{\psi u_t^{\rho_1}}{(1-u_t)^{\rho_2}} \right)^{\frac{1}{(1-\rho_1)}} = w_t^u \tag{2.7}$$

$$u_t w_t^u = c_t^u \tag{2.8}$$

Similar to the skilled labor problem, (2.7) gives the trade-off between consumption and working hours of unskilled labor, which is determined by wage of unskilled labor, w^u, inter-temporal elasticity of consumption and leisure, and weight of leisure in utility function. However, there is no Euler equation obtained for unskilled labor since there is no capital accumulation or investment decision for unskilled worker. Thus, she does not make an inter-temporal decision on consumption, she only consumes what she earns from supplying her labor in each period.

Firm's Problem:

In a perfectly competitive market, firm solves profit maximization problem subject to a different type of technology, in which physical capital is more complementary to skilled labor than to unskilled labor. In other words, elasticity of substitution between physical capital and unskilled labor, $\xi_{k,u}$, is greater than the elasticity of substitution between physical capital and skilled labor,

$\xi_{k,s}$, i.e. $\xi_{k,u} > \xi_{k,s}$.

For general production technologies with more than two inputs there is no single definition for the elasticity of substitution between two inputs. In this analysis Allen-Uzawa partial elasticity of substitution is used to determine the parameter values needed to have capital-skill complementarity in production technology.[4]

Following Duffy *et al.* (2004), using Allen-Uzawa partial elasticity of substitution, if $\frac{1}{1-v} > 1$, i.e. $0 < v < 1$, then $\xi_{k,u} > \xi_{k,s}$, there is capital-skill complementarity.[5] The technology with capital-skill complementarity has the following form:

$$f(k_t, s_t, u_t) = e^{z_t} a[\theta {k_t}^v + (1-\theta)u_t^v]^{\alpha/v} s_t^{1-\alpha} \tag{2.9}$$

Similar to the Cobb-Douglas case, z_t is the technology shock that evolves according to the same AR(1) process in equation 2.4). First order conditions of this problem are:

$$f_k(k_t, s_t, u_t) = r_t$$

$$f_s(k_t, s_t, u_t) = w_t^s$$

$$f_u(k_t, s_t, u_t) = w_t^u$$

where

$$f_k(k_t, s_t, u_t) = \frac{\theta \alpha k_t^{v-1} f(k_t, s_t, u_t)}{[\theta k_t^v + (1-\theta)u_t^v]}$$

$$f_u(k_t, s_t, u_t) = \frac{(1-\theta)\alpha u_t^{v-1} f(k_t, s_t, u_t)}{[\theta k_t^v + (1-\theta)u_t^v]}$$

[4]Duffy *et al.* (2004) show that using the definition of Allen-Uzawa partial elasticity of substitution gives the same conditions for parameters as those of direct partial elasticity of substitution.

[5]v takes different values and it is always assumed to be between 0 and 1 in the simulation.

$$f_s(k_t, s_t, u_t) = \frac{(1-\alpha)f(k_t, s_t, u_t)}{s_t}$$

2.4 Single Agent Model with Capital Adjustment Cost

Here, the standard RBC model with Cobb-Douglas technology is extended by introducing capital adjustment cost in the budget constraint of household. Specifying capital adjustment cost function $g(I_t) = \frac{\gamma}{2}(I_t - I^{ss})^2$:[6]

$$max\ E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{c_t^{1-\rho_1}}{1-\rho_1} + \psi \frac{(1-l_t)^{1-\rho_2}}{1-\rho_2} \right]$$

subject to

$$k_{t+1} = (1-\delta)k_t + I_t$$

$$c_t + I_t + g(I_t) = f(k_t, l_t)$$

The first order conditions of the problem are as follows:

$$\psi \frac{c_t^{\rho_1}}{(1-l_t)^{\rho_2}} = w_t$$

$$\left(\frac{c_{t+1}}{c_t}\right)^{\rho_1} = \beta \left[(1-\delta) + \frac{r_{t+1}}{[1+\gamma(I_t - I^{ss})]} \right] \tag{2.10}$$

The Euler equation (2.10) implies that if the investment undertaken by the household is away from its steady-state level, she prefers to consume much today as compared to the case in which there is no capital adjustment cost

[6] $g(I^{ss}) = 0$ and $g'(I^{ss}) = 0$.

while making inter-temporal consumption decision. Firm's problem is exactly the same as in the first model.

2.5 Two-Agent Model with Capital Adjustment Cost

Here, the standard RBC model with capital-skill complementarity is extended by introducing capital adjustment cost in the budget constraint of household. Since all the capital is owned by skilled labor and she is the only type of agent making investment decision, the problem of unskilled labor does not change. Skilled labor's problem becomes:

$$max\ E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{c_t^{s(1-\rho_1)}}{1-\rho_1} + \psi \frac{(1-s_t)^{(1-\rho_2)}}{1-\rho_2} \right]$$

subject to

$$k_{t+1} = (1-\delta)k_t + I_t$$

$$c_t^s + I_t + \frac{\gamma}{2}(I_t - I^{ss})^2 = w_t^s s_t + r_t k_t$$

The first order conditions are as follows:

$$\psi \frac{(c_t^s)^{\rho_1}}{(1-s_t)^{\rho_2}} = w_t^s$$

$$\left(\frac{c_{t+1}^s}{c_t^s} \right)^{\rho_1} = \beta \left[(1-\delta) + \frac{r_{t+1}}{[1+\gamma(I_t - I^{ss})]} \right] \tag{2.11}$$

Similar to the single agent model with Cobb-Douglas technology, the Euler equation (2.11) implies that if the investment undertaken by the skilled labor is away from its steady-state level, she prefers to consume much today as compared to the case in which there is no capital adjustment cost while making inter-temporal consumption decision. Problems of firm and unskilled labor are exactly the same as the previous two agent model without investment adjustment cost.

CHAPTER 3

SIMULATION

Since the models can not be solved analytically, I simulate the model solving dynamic stochastic general equilibrium model.[1] Before going into the details of the simulation results, I will explain the parameter values of the models used in the simulation.

3.1 Parameters

Since the analysis is based on the comparison of the models in terms of persistence difference of the models, the same parameter values will be used in all of the models. Considering each period of the simulation as a quarter, I set the depreciation rate to be 0.025 ($\delta = 0.025$) and the discount rate be 0.99 ($\beta = 0.99$). I set the capital income share equal to 1/3 ($\alpha = 1/3$), which is the standard value for the US data. Finally, I set exogenous technology parameter, a, to be 1 since it only affects the scale of the economy.

One of the common utility forms used in standard RBC models is log-linear utility function, i.e. $u(c, l) = log(c) + \psi(1 - l)$, which corresponds to the parameter values $\rho_1 = 1$ and $\rho_2 = 0$.[2] Since this form leads to constant labor supply of unskilled labor in the models having technology with capital-skill complementarity, instead of setting $\rho_1 = 1$, I will set $\rho_1 = 0.9$. Hence, the

[1] I use the program DYNARE. One can find the details of this program in Juilliard (1996).
[2] See King *et al.* (1988).

Table 2: Parameter Values

α	β	δ	a	ρ_1	ρ_2	ψ	σ	$st.dev.(\epsilon)$	θ	v	γ
0.33	0.99	0.025	1	0.9	0	4.5	0.70	0.0072	0.4	0.5	2.2

utility function of consumption with these parameter values will have similar inter-temporal elasticity of consumption with the logarithmic case. Moreover, I set the ψ equal to 4.5 in order to match the steady state value of hours worked l, which is about 20% of available time ($l^{ss} = 0.20$).

The standard deviation of the AR(1) process of the stochastic shock is determined according to the Solow residual, which is estimated as 0.0072 for quarterly data set.[3] I set the persistence parameter, σ, equal to 0.70 to match the persistence of output in standard RBC model since the aim of the paper is to see whether there is an improvement in persistence of output with capital-skill complementarity.

The adjustment cost parameter, γ, is chosen regarding the paper of Cogley and Nason (1995), which have a similar form of adjustment cost function in their model, i.e. $\gamma = 2.2$.[4]

The parameter values of production function having capital-skill complementarity is set following the study of Stokey (1996). The parameter that determines the capital share of capital, θ, is set to be equal to 0.4 and the parameter that determines the elasticity of substitution between capital and unskilled labor, v, is chosen as 0.5 so that the production function has the property of capital-skill complementarity according to the definition of Allen-Uzawa partial elasticity of substitution. Furthermore, I will change the values of these two parameter values in the simulation in order to see the effects of them on the models.[5]

[3] See King and Rebelo (2000).

[4] Actually, they evaluated the adjustment cost parameter ($\gamma = 2.2$) using the calibration results of Shapiro (1986).

[5] The different values that are set for parameter v is between 0 and 1 so the production

3.2 Main Results

This paper analyzes the effect of using a different type of production function with capital-skill complementarity on RBC models. That is, it tries to figure out whether two-agent models having production technologies with capital-skill complementarity succeed to capture the persistence of an exogenous technology shock within RBC framework. I will analyze the impulse response functions of the models described above and compare the models with Cobb-Douglas production function vs the ones with capital-skill complementarity. Firstly, I will analyze the standard RBC model without capital adjustment cost and then I will present the results of the extended model with adjustment cost.

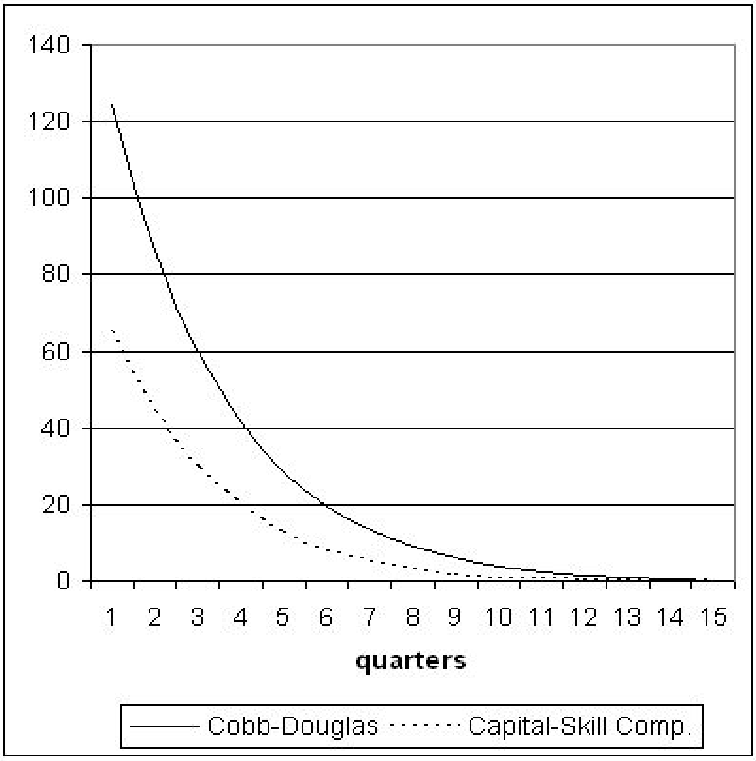

Figure 1: The Impulse Responses of Output in Models with Cobb-Douglas and Capital-Skill Complementarity

Figure 1 shows the impulse response functions of output obtained from the single agent model having Cobb-Douglas technology and two agent model having production technology with capital-skill complementarity, given the same

function still has the property of capital-skill complementarity according to the definition of Allen-Uzawa partial elasticity of substitution.

positive technology shock. As seen in the Figure 1, in both models output responds to the shocks in the same direction, but in different manner quantitatively. In fact, the output in model having production technology with capital skill complementarity responds less than that of model with Cobb-Douglas production function does. Actually, Cobb-Douglas production function and the production function with capital-skill complementarity are not completely different functions. Cobb-Douglas production function is a particular type of the latter one with certain parameter values.[6] Therefore, disparity in impulse responses of those models stemps from the different parameter values of v ($v = 0.5$) and θ ($\theta = 0.4$). Therefore, it is better to compare the impulse response functions by changing one of this parameter values ceteris paribus.

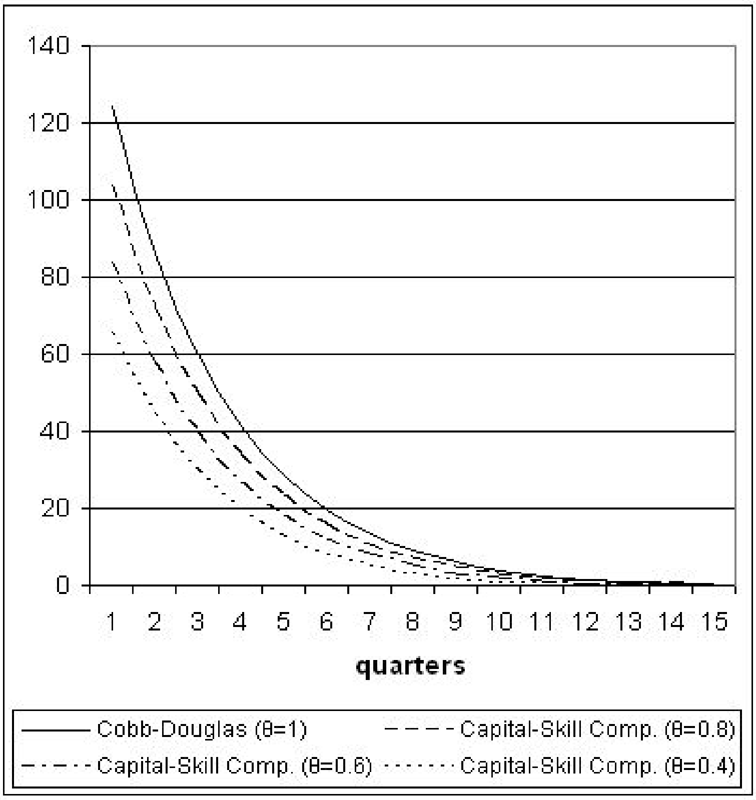

Figure 2: The Impulse Responses of Output in Models with Different θ Values

Remember that θ refers to the factor share of physical capital in our production function. As θ goes to 1 the production function behaves more like

[6]When $v = 1$ and $\theta = 1$, they are identical.

Table 3: Persistence Coefficients (for different θ values)

Model	C-D ($\theta = 1$)	CSC ($\theta = 0.8$)	CSC ($\theta = 0.6$)	CSC ($\theta = 0.4$)
AC	0.693	0.689	0.684	0.672

Notes: C-D: Model with Cobb-Douglas production function. CSC: Model having production function with Capital-Skill Complementarity.

Cobb-Douglas production function since the factor share of physical capital increases in the production function. As θ increases, the effect of technological shock on impulse response function of output becomes larger.[7] The reason is that as the factor share of physical capital increases the physical capital is affected more by the given positive shock opposed to working hour of unskilled labor (u). Moreover, as the physical capital is an accumulated variable, the shock is more persistent compared to low values of θ.

The persistence difference of these models is not clear from figures so I evaluate the AR(1) coefficient of the impulse response series of output. Table 3 shows the coefficients corresponding to the different θ values. As it is suggested, the coefficient decreases as θ decreases.

In the production function having capital-skill complementarity, $\frac{1}{1-v}$ is the intraclass elasticity of substitution between physical capital (k) and unskilled labor (u). Therefore, as v increases, k and u are relatively better substitutes. Contrary to the expectation, for larger values of v, the production function having capital-skill complementarity behaves more likely as Cobb-Douglas production function with two factors does. The contribution of the capital-skill complementarity on Cobb-Douglas production function is not that the physical capital and unskilled labor are substitutes but they are *good* substitutes. In other words, as v gets smaller k and u become good substitutes from being *perfect* substitutes.[8] On the other extreme, when $v = 0$ the production func-

[7]See Figure 2.

[8]Here, the intraclass elasticity of substitution is considered. Since the factor share of physical capital is 1 in the Cobb-Douglas production technology, the elasticity of substitution between physical capital and unskilled labor is not greater than that of between physical

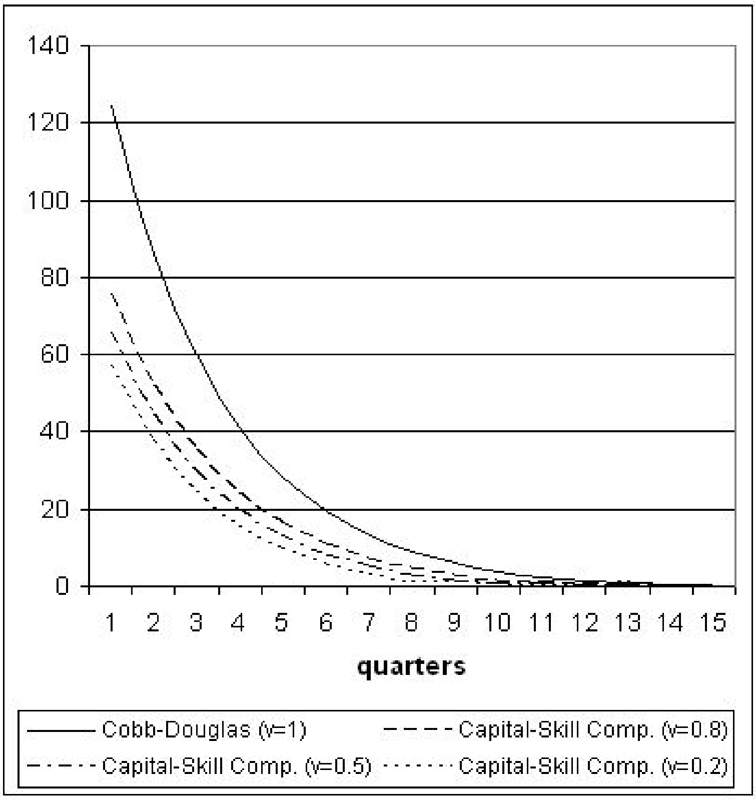

Figure 3: The Impulse Responses of Output in Models with Different v Values

tion changes into Cobb-Douglas in three factors (k, s, and u). As it is seen in Figure 3 as the value of v decreases the effect of technological shock on the impulse response of output gets smaller. The reason is that as v decreases the physical capital and unskilled labor becomes more complementary. Therefore, the effect of positive technological shock on unskilled labor gets larger while the effect on physical capital becomes smaller compared to the higher values of v. Since the physical capital is an accumulated variable, the positive technological shock is less persistent for low values of v regarding the impulse response functions of output. Since the persistence difference of the impulse responses of output is not clear from Figure 3, I also evaluate the AR(1) coefficient of the impulse response series of output for these models. Table 4 displays the AR(1) coefficients of impulse response functions corresponding to the different

capital and skilled labor using Allen-Uzawa partial elasticity of substitution. So, the fact that physical capital and unskilled labor are perfect substitutes does not mean that there is capital-skill complementarity in Cobb-Douglas technology.

Table 4: Persistence Coefficients (for different v values)

Model	C-D ($v = 1$)	CSC ($v = 0.8$)	CSC ($v = 0.5$)	CSC ($v = 0.2$)
AC	0.693	0.685	0.672	0.654

Notes: C-D: Model with Cobb-Douglas production function. CSC: Model having production function with Capital-Skill Complementarity.

v values. As it is suggested, the coefficient decreases as v decreases, i.e. the technological shock on output is less persistent as capital and skilled labor becomes less substitute.

3.3 The Effect of Capital Adjustment Cost on Impulse Response Functions

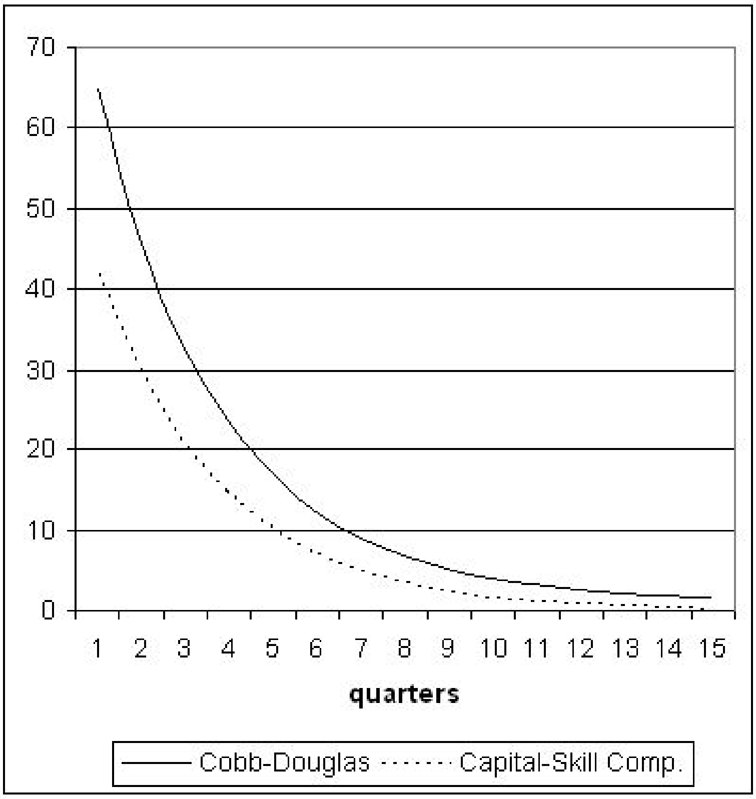

Figure 4: The Impulse Responses of Output in Models with Capital Adjustment Cost

Table 5: Persistence Coefficients of the Model with Adjustment Cost (for different parameter values)

Model	C-D ($\theta = 1$)	CSC ($\theta = 0.8$)	CSC ($\theta = 0.6$)	CSC ($\theta = 0.4$)
AC	0.714	0.712	0.708	0.699

Model	C-D ($v = 1$)	CSC ($v = 0.8$)	CSC ($v = 0.5$)	CSC ($v = 0.2$)
AC	0.714	0.707	0.699	0.687

Notes: C-D: Model with Cobb-Douglas production function. CSC: Model having production function with Capital-Skill Complementarity.

Recall that the investment adjustment cost of the model is $g(I(t)) = \frac{\gamma}{2}[I(t) - I^{ss}]^2$. Since the positive technological shock leads to an increase in both consumption and investment, adding adjustment cost to the model will lead to a rise in the cost of increasing the investment. Thus, the effect of technological shock on investment and physical capital is smaller compared to that of the model without adjustment cost. However, once the amount of physical capital has increased, since it is more costly to change the capital level because of the adjustment cost, the persistence of output in the model with adjustment cost is higher than that of the standard RBC model. This finding is consistent with the studies in literature.[9]

Considering endogenous propagation mechanism of technology shocks, extension of standard RBC models by using production function with capital-skill complementarity yield no improvement with reference to capital adjustment cost.

[9]The persistence difference between the models with and without adjustment cost can easily been observed by comparing the auto correlation coefficients in Table 3 and 4 with those in Table 5.

CHAPTER 4

CONCLUSION

The aim of this paper is to analyze the persistence of output in standard RBC models using a production function with capital-skill complementarity instead of using the standard Cobb-Douglas production function.

Capital-skill complementarity is mostly used in open economy models to study the effect of openness on wage inequality. I studied whether this production function fits the better known stylized facts in closed economy frameworks.

To compare the level of propagation of technology shocks I simulate the models using both Cobb-Douglas technology and a production function with capital-skill complementarity with the same parameter values. As a result, I find that adding capital-skill complementarity worsens the propagation of shocks in the standard RBC models with and without capital adjustment costs, that is, it leads to even lower endogenous propagation in a model that already lacks sufficient propagation. The main reason is that exogenous shocks affect the physical capital, which is one of the main component that leads to endogenous propagation mechanism, less in models having technology with capital-skill complementarity compared to the those having Cobb-Douglas technology since factor share of physical capital and intraclass elasticity of substitution between physical capital and unskilled labor is less in the production function with capital-skill complementarity than those with Cobb-Douglas technology.

Hence, using production functions with capital-skill complementarity in open economy models should not be thought of as an innocuous assumption. This device, while helping the models generate skill premiums observed in the data, causes the fit of the model to some basic macroeconomic quantities to worsen. It is an open question whether the tradeoff between better fit to wage premium and worse performance in propagation is attractive enough to trust the conclusions of models using this production function.

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